# CoinJoin in the Wild 

(Full Version)

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#### Abstract

CoinJoin is the predominant means to enhance privacy in non-private cryptocurrencies, such as Bitcoin. The basic idea of CoinJoin is to create transactions that combine equal-valued coins of multiple users. This mixing of coins aims to prevent linkage of the users' transactional in- and outputs. The cryptocurrency Dash employs a built-in CoinJoin service and, therefore, is ideal for empirically studying CoinJoin. This paper presents the first empirical analysis of Dash, which reveals that over $40 \%$ of all private transactions can be de-anonymized depending on underlying assumptions. The main issue of these attacks is the coin-aggregation problem, i.e. the need to combine outputs of several CoinJoin transactions. The coin aggregation problem is not specific to Dash and affects other cryptocurrencies as empirical evidence in Bitcoin suggests. We show that the logical solution to the problem, namely CoinJoin transactions with non-fixed arbitrary values, suffers from other privacy weaknesses. We propose a novel mixing algorithm to mitigate the need for coin aggregation without introducing additional privacy vulnerabilities. In contrast to prior mixing algorithms, our approach removes the need for fixed values by dynamically creating equal-valued CoinJoin transactions. The mixing algorithm is not specific to Dash, and integration into other cryptocurrencies, especially into Bitcoin, is possible.


Keywords: anonymous transactions, linking heuristics, de-anonymization, mixing

## Table of Contents

CoinJoin in the Wild ..... 1Dominic Deuber and Dominique Schröder
1 Introduction. ..... 2
2 Preliminaries ..... 4
3 Dash ..... 5
4 Empirical Anonymity Analysis ..... 6
5 Enhancing Privacy of Mixing. ..... 10
A Differences in the Analysis in Bitcoin ..... 14
B Limitations to Arbitrary-Value Mixing ..... 15

## 1 Introduction

More and more it seems as if cryptocurrencies have come to stay and are not mere hype. The most widely used cryptocurrency Bitcoin bit is often perceived to provide anonymity. However, Bitcoin is not anonymous as it is possible to link addresses that belong to the same user $\mathrm{RS}^{2} \mathrm{AKR}^{+} 13 \mathrm{MPJ}^{+} 13$. The goal of mixing protocols is the prevention of these linkage attacks. CoinJoin Max13] is the most widely used protocol. It combines inputs and outputs from multiple users and creates a random permutation that hides the correlation between input and output addresses and, thus, between users. Even though Monero [mon] and Zcash [zca] are two cryptocurrencies that achieve privacy by design, it is crucial to study and improve CoinJoin for two reasons. First, Bitcoin is still the most commonly used cryptocurrency, especially in the dark web Eur, and supports CoinJoin to improve privacy without requiring to swap coins to a more privacy-preserving currency. Second, Monero and Zcash are already banned in South Korea Ike, and there is a risk that other jurisdictions will follow. If privacy-preserving currencies are banned, CoinJoin on top of Bitcoin is among the only possibilities for people in the corresponding jurisdictions to add privacy to their cryptocurrency activities.

The cryptocurrency Dash dasa employs a built-in CoinJoin mechanism. Dash has not been researched before, although Dash has a market capitalization of over four billion USD, is the second-largest cryptocurrency with built-in privacy features, and the third most established privacy coin for transactions in the dark web Eur. By the beginning of 2021 Dash's blockchain accounted for approximately 1.4 million blocks including over 31 million transactions.

### 1.1 Empirical Analysis of Anonymity

We present the first empirical analysis of anonymity in Dash that combines new and existing attacks to evaluate Dash's anonymity level and gain insights on CoinJoin and its privacy. We introduce a novel attack that we call Backlink attack. In essence, Backlink attack carefully combines multi-input heuristic $\left[\right.$ RS13 AKR ${ }^{+} 13 \mathrm{MPJ}^{+} 13$ ] and a newly developed heuristic to find address clusters.

We put forward the DC attack, which is a modification of the cluster-intersection attack according to Goldfeder et al. GKRN18]. The DC attack revealed a fundamental problem with CoinJoin, namely the coin aggregation problem. As Dash uses fixed values in their CoinJoin transactions, users generally need to aggregate coins of several CoinJoin transactions that fuel the attack. To ascertain whether the coin aggregation problem is also present in other cryptocurrencies, we analyze the impact of our attacks for Bitcoin. Results: It was found that $15.1 \%$ of non-mixing transactions that spend private coins are linkable by the Backlink attack. In terms of address clusters, applying the newly developed heuristic reduces the number of clusters by almost two-third compared to only applying the multi-input heuristic. By applying the DC attack, we were able to link over $40 \%$ of Dash's private transactions depending on the underlying assumptions of the attack.

In Bitcoin, around $23 \%$ of all transactions which spend outputs from a CoinJoin transactions contain backlinks. In addition, more than one-tenth of all transactions do so from CoinJoin transactions spend from at least two different CoinJoin transactions, which indicates that coin aggregation is also a problem in Bitcoin.

### 1.2 Cookie Monster Mixing

Our analysis suggests that the privacy issues in Dash result from the fact that Dash only supports equalvalued mixing with fixed values and allows users to combine their coins in a way that might de-anonymize them. We analyze arbitrary-value CoinJoin as proposed by Maurer et al. MNF17] and show that it has other privacy weaknesses, which is why it is not a suitable way to solve the coin aggregation problem. To remove the issue of only fixed values being mixed without introducing additional privacy vulnerabilities, we propose a novel mixing algorithm that we call Cookie Monster Mixing. The algorithm is inspired by the cookie monster problem BK15] and removes the need to split and combine coins before and after mixing. Thus, the information that multiple coins of different mixing transactions belong together is no longer present on-chain. As a consequence, cluster-intersection attacks without additional off-chain information are no longer possible. We have formalized the problem as an integer quadratic problem and propose an efficient greedy algorithm to solve it. A prototype implementation reinforces the practical efficiency. Through experimentation, we have validated that the greedy algorithm is nearly optimal.

### 1.3 Responsible Disclosure

We reported our findings to the Dash Core Group, one of the organizations working for the Dash network, and declared our willingness to support the implementation of the suggested countermeasures. With Dash Core Release 0.16.0.1 dasc, Dash has implemented some of our suggested countermeasures to improve privacy.

### 1.4 Related Work

A major concern of CoinJoin is that the users need to trust an external mixing service that creates the transaction. Alternative approaches to mitigating this weakness have been proposed, such as CoinShuffle RMK14] or its more efficient successor CoinShuffle++ RMK17. For Ethereum, a trustless tumbler Möbius has been presented that achieves mixing through a smart contract based on ring signatures and stealth addresses MM18.

While CoinJoin is a mixing service that can be used as an extension of traditional cryptocurrencies, new privacy-preserving cryptocurrencies have also evolved, spearheaded by Monero [mon] and Zcash [Zca]. Monero is based on the CryptoNote protocol VS13] and mainly uses ring-confidential transactions [NML16] to achieve privacy. Conversely, Zcash is based on the Zerocash protocol $\left[\mathrm{BCG}^{+} 14\right]$ and mainly uses zero-knowledge, succinct, non-interactive arguments of knowledge to achieve privacy. The anonymity of both cryptocurrencies has since then been subject to analyses KFTS17 MSH ${ }^{+}$18 KYMM18 Que17.

Goldfeder et al. GKRN18] showed that CoinJoin transactions in Bitcoin are vulnerable to the so-called cluster-intersection attack. Kalodner et al. $\left.\mathrm{KGC}^{+} 17\right]$ experimentally validated the applicability of the clus-ter-intersection attack to Dash on simulated transactions. In contrast, we apply the attack to the entire Dash blockchain data. To do so, however, we needed to refine it, as there are several underlying assumptions to take into account.

The major services for CoinJoin in Bitcoin are Wasabi Wallet [was], Samourai Wallet [sam] and JoinMarket joil. All distinguish between pre- and post-mixing. However, Wasabi and Samourai require fixed output values and thus might benefit from the flexibility Cookie Monster Mixing provides in building their CoinJoin transactions. JoinMarket allows for flexible output values, albeit in a different setting. Its protocol distinguishes between takers and makers where the taker pays the makers to participate in the mixing.

## 2 Preliminaries

In this section, we briefly explain concepts necessary to understand our attacks and countermeasures. We introduce transactions, the multi-input heuristic and CoinJoin followed by a high-level description of the cluster-intersection attack.

### 2.1 Transaction



Fig. 1: Transaction

A transaction consists of a list of inputs and outputs. In simple terms, an output comprises an amount of a given cryptocurrency $C C$ and the hash $h_{p k}$ of a public key $p k$, which is also called an address. Inputs are references to outputs of previous transactions. A transaction with two inputs and three outputs is depicted in Figure 1a. The two inputs refer to the outputs at indices out $i_{i d_{1}}$ and out $i_{d_{2}}$ of transactions with hashes $t x_{h_{\text {ash }}^{1}}$ and $t x_{h a s h_{2}}$ respectively. Each output $o_{i}$ for $i \in[a, b, c]$ of the transaction specifies an address $h_{p k_{i}}$ and an amount $\# C C_{i}$ of the cryptocurrency. To spend an output $o_{i}$ of this transaction in a succeeding transaction, a public key $p k$ must be provided whose hash equals $h_{p k_{i}}$ and a signature that verifies for $p k$. It is common for a transaction to have multiple in- and outputs, as the input value needs to be spent completely. For example, if the inputs amount to $5 C C$ but the user only wants to spend $4 C C$ to $h_{p k_{a}}$ and $h_{p k_{b}}$, they will create an output $o_{c}$ to send back the remaining $1 C C$ to an address they control $\left(h_{p k_{c}}\right)$, which is also called change (address). Outputs that can be referenced by a transaction, but have not yet been, are called unspent-transaction outputs.

### 2.2 Multi-Input Heuristic

If a transaction has multiple inputs, the following address-linking heuristic can be applied.
Heuristic 21 (multi-input heuristic [RS13 $\mathbf{A K R}^{+} \mathbf{1 3} \mathbf{\mathbf { M P J } ^ { + } \mathbf { 1 3 } ] ) \text { All addresses referred to in the inputs }}$ of a transaction are controlled by the same entity.

The reason is that the computation of the signature of each input requires the knowledge of the secret key. Other heuristics take advantage of the fact that coins can only be spent in their entirety with the spending user usually sending back the remaining amount of the cryptocurrency to a change address they control [RS13 $\left.\mathrm{AKR}^{+} 13 \mathrm{MPJ}^{+} 13\right]$. Linking addresses results in sets of addresses, so-called address clusters, which are likely controlled by the same entity.

### 2.3 CoinJoin

The basic idea of CoinJoin Max13 is special CoinJoin transactions, which combine the in- and outputs of multiple users. An example of such a transaction is shown in Figure 1b. The transaction has three inputs and three outputs from three different users $A, B$ and $C$. Here, we assume that all in- and outputs have
the same value. By merely examining the transaction it is not possible to determine which input $i_{x}$ for $x \in[A, B, C]$ belongs to which output $o_{y}$ for $y \in[1,2,3]$. As the inputs are controlled by three different users, the multi-input heuristic (Heuristic 21) cannot be applied.

### 2.4 Cluster-Intersection Attack

Goldfeder et al. GKRN18] showed that CoinJoin transactions in Bitcoin are vulnerable to the so-called cluster-intersection attack, which works as follows. For each output of a CoinJoin transaction, its anonymity set is determined by inspecting the inputs of the transaction as, ideally, each input could be the origin of each output. The anonymity set contains all possible address clusters that might be the output's origin. Additional information may likely reveal that the same entity controls certain outputs of different CoinJoin transactions. In that case, the corresponding anonymity sets can be intersected, i.e. address clusters that are present in all sets can be identified. If there is only one address cluster in the intersection, this cluster might be the origin of those outputs. Additional information revealing that the same entity controls certain outputs of different CoinJoin transactions can, for instance, be a single transaction spending such outputs. Then, the information follows from the multi-input heuristic (Heuristic 21) and thus is on-chain information. Furthermore, it is also possible that the payment recipient can derive the information off-chain, as seen in the following example. Imagine a merchant receives two payments from the same customer, and each of the payments is the output of a different CoinJoin transaction. If the anonymity sets of both outputs only have a single address cluster in common, the merchant can assume that this cluster belongs to the customer.

## 3 Dash

In this section, we introduce Dash and explain how it addresses privacy in its PrivateSend feature as a necessary prerequisite for our attacks in Section 4

### 3.1 Overview

The cryptocurrency Dash, having forked from Bitcoin in 2014 dasb, follows the same basic structure: the decentralized transaction ledger is maintained in a peer-to-peer network that uses a consensus mechanism to agree on new transactions. The transactions are organized in blocks and the ledger is often called blockchain; the nodes in the consensus mechanism are called miners. They are rewarded for their participation through block rewards, i.e., newly generated units of the cryptocurrency and transaction fees. Dash differs from Bitcoin mainly by implementing a native CoinJoin feature, PrivateSend, and a feature that reduces the time it takes until a transaction can be considered final, InstantSend. Both features are achieved by so-called masternodes, which are special nodes participating in the Dash network. In contrast to miners, masternodes do not directly participate in the consensus mechanism but mainly provide PrivateSend and InstantSend as a service. They are rewarded for their services with fees. Additionally, they also receive parts of the block rewards in so-called CoinbaseTXs. InstantSend solves the problem of confirmation time that is present in Bitcoin. To do so, a quorum of masternodes locks the inputs of a proposed transaction, which leads to competing transactions being rejected dase. We do not consider InstantSend in the rest of the paper, as we are concentrating on privacy.

### 3.2 PrivateSend

PrivateSend is a service provided by masternodes to prevent the linkage of addresses from different transaction outputs potentially belonging to the same entity. Put in simple terms, PrivateSend implements CoinJoin (see Section 2.3). There are several services that support the process of finding other users to group with in order to build a CoinJoin transaction. In an ideal scenario, only these services learn the input-output mapping, i.e., the mapping of inputs to corresponding outputs.


Fig. 2: PrivateSend mixing procedure
The mixing process of Dash is depicted in Figure 2 and works as follows: a user's wallet splits the value of some unspent-transaction outputs in a CreateDenomTX, and sends it to the network (1). This step is a necessary prerequisite, as mixing in Dash requires equal and fixed values. Next, the wallet reports a mixing request to a randomly selected masternode (2). This request includes certain unspent-transaction outputs of the CreateDenomTX as inputs and equally as many outputs with addresses that the wallet controls. If enough other users (dashed lines) also reported their request to the masternode, it builds a MixingTX (3), consisting of all of the users' input-output pairs. At this point, the masternode reports the MixingTX back to each user's wallet (4), such that it can sign the inputs. Before doing so, the wallet ensures that the outputs initially reported in its request are contained in the list of outputs of the MixingTX. This check is crucial in guaranteeing that the masternode cannot steal any coins by replacing outputs with its own. If each wallet only signs so long as the check is successful, the masternode cannot redirect money to their addresses since the sum of the input values must exactly match the sum of the output values in MixingTXs. The reason is that the fees required for mixing are decoupled from the MixingTXs and therefore omitted for the sake of simplicity. After each wallet has signed their inputs and sent the signatures to the masternode (5), the masternode can send the MixingTX to the network (6). Each wallet then has private unspent-transaction outputs, which can be used as inputs for further mixing rounds or spent in PrivateSendTXs, the final transaction type used in PrivateSend. A PrivateSendTX is a transaction, whereby the wallet implementation ensures that it only spends private unspent-transaction outputs from MixingTXs.

## 4 Empirical Anonymity Analysis

In this section, we analyze the anonymity provided by CoinJoin in the context of Dash and Bitcoin. For the analysis of Dash, we ran a Dash full node, version 0.16.1.1 dasc and build an analysis pipeline using BlockSci KML ${ }^{+} 20$ with version 0.5.0. First, we retrieved the raw blockchain data up to December 31, 2020, which corresponds to a chain of 1397530 blocks. Then we detected the type of transactions that are relevant for our attacks. In the backlink attack, we linked address clusters based on the multi-input heuristic (see Heuristic 21) and a new clustering heuristic. Finally, we refined the cluster-intersection attack by adding false-positive rejection mechanisms and addressing uncertainty about its underlying assumptions to make the attack applicable to Dash (DC attack). The differences of our analysis of Bitcoin are stated in Appendix A

### 4.1 Transaction Type Detection

We ran a transaction type detection algorithm for the identification of relevant transactions for PrivateSend. This algorithm processes the data retrieved from our full node, and it takes advantage of the fact that the mixing denominations in Dash are of the form $1.00001 \times 10^{k}$ for $k \in[-3, \ldots, 1]$. Due to this structure, it seems unlikely that it would not be a mixing denomination if we were to encounter such a value. As a consequence, our detection mechanism should produce few to no false positives. By design of our detection mechanism, a transaction can only have one type, i.e., there is no ambiguity. We consider each transaction that does not
match any of the following types to be an OtherTX. MixingTX is a transaction with equally many inputs and outputs, all with the same denomination. This is due to the fact that the fee is decoupled from mixing. Thus, there is exactly one output for each input. We additionally require that there are at least three inputs as at least three participants are required for mixing (see Dash's whitepaper [DD]). CreateDenomTX is a transaction that is not a MixingTX if there are at least two outputs, while one output needs to have one of the mixing denominations. Furthermore, we allow at most two non-mixing-denominated outputs since one of them might be the change output and thus of arbitrary value. The other might be a special output required to pay mixing fees. PrivateSendTX is a transaction that is not a MixingTX if it has more than one input and all the inputs are mixing denominations. However, we only consider it a PrivateSendTX if it has exactly one output since a PrivateSendTX does not allow change dasg.

In Figure 3 the transactions are listed by their type, where the total number of transactions was 31563841 . Only $0.4 \%$ (110 846) of all transactions are PrivateSendTXs.
Bitcoin In Bitcoin, we found that out of 493118000 transactions, 1767452 ( $0.4 \%$ ) were CoinJoinTXs (the counterpart of Dash's MixingTXs). For the transactions entering into and spending from CoinJoinTXs, we detected 5865534 (1.2\%) PreCoinJoinTXs and 7228843 (1.5\%) PostCoinJoinTXs.

| CoinbaseTXs | $4.4 \%$ |
| :--- | ---: |
| PrivateSendTXs | $0.4 \%$ |
| MixingTXs | $10.1 \%$ |
| CreateDenomTXs | $1.5 \%$ |
| OtherTXs | $83.6 \%$ |

Fig. 3: Dash transaction types


Fig. 4: Backlink analysis

### 4.2 Backlink Attack

We introduce the Backlink attack, which directly links addresses occurring in the output of a MixingTX, i.e., linking them to output addresses of a CreateDenomTX. There are transactions in Dash that spend outputs of MixingTXs and at the same time outputs of CreateDenomTXs. We call such a transaction a BacklinkTX and the output of the CreateDenomTX a backlink. Such a transaction is shown in Figure 4 The transaction's first input is a reference to the fourth output of the CreateDenomTX, which is the backlink ${ }^{1}$ Thus, as a direct result of the multi-input heuristic (see Heuristic 21), the addresses of the MixingTX, $h_{p k_{c}}$ and $h_{p k_{d}}$, are linkable, as under the assumptions of the multi-input heuristic there is a link to $h_{p k_{4}}$, which is an output address of the CreateDenomTX.

However, the linkable addresses can be further linked as all input and all output addresses of a CreateDenomTX are most likely controlled by the same entity. The reason for this is that CreateDenomTXs are generated by a user's wallet. This leads to the following address clustering heuristic:

Heuristic 41 All in- and output addresses of a CreateDenomTX are controlled by the same entity.

[^0]As a result, $h_{p k_{c}}$ and $h_{p k_{d}}$ of the BacklinkTX in Figure 4 can not only be linked to $h_{p k_{4}}$ (multi-input heuristic, dashed line) but also to $h_{p k_{1}}$ to $h_{p k_{3}}$ and to the address corresponding to the output of $t x_{h a s h}$ at index out ${ }_{i d}$ (Heuristic 41 dotted line). Note that reasonable clustering results are only achieved by combining both heuristics. Applying the multi-input heuristic would allow linking to the backlink. However, without Heuristic 41, the backlink address would, in general, be in a single cluster and not reveal any additional information about the user's transaction history before mixing.

To detect backlinks, we do the following. We first iterate over all transactions. Then, we check each transaction as to whether it has inputs referencing MixingTXs as well as inputs referencing CreateDenomTXs. To identify the corresponding clusters, we add our heuristic to the clustering module of BlockSci, which already implements the multi-input heuristic.

We found that out of the 174834 transactions that are not MixingTXs but spend mixing outputs, 26402 (15.1\%) have backlinks. In terms of addresses from the outputs of MixingTXs, we found that out of 6833911 addresses, $836230(12.2 \%)$ are linkable. Applying only the multi-input heuristic resulted in 23580 clusters. We reduced that number by almost two thirds by additionally applying our Heuristic 41, which resulted in only 7920 clusters.
Bitcoin The attack slightly differs in Bitcoin as there are no explicit CreateDenomTXs. Instead of CreateDenomTXs, we consider the PreCoinJoinTXs. Thus, a BacklinkTX in Bitcoin is a PostCoinJoinTX with at least one input from a PreCoinJoinTX. We found that out of 7228843 PostCoinJoinTXs, 1674070 ( $23.2 \%$ ) have backlinks. This shows that backlinks are also present and even more problematic in Bitcoin.

### 4.3 DC Attack

We introduce the DC attack as a modification of the cluster-intersection attack (Section 2.4). First, we give a high-level description of the cluster-intersection attack in the context of Dash, followed by a discussion of which modifications are necessary to apply the attack. Finally, we present the Dash cluster-intersection attack (DC attack).


Fig. 5: Cluster intersection in Dash
An overview of the cluster-intersection attack in Dash is depicted in Figure 5. The PrivateSendTX has inputs from both MixingTX 1 and MixingTX 2. If we trace back the inputs, the set of CreateDenomTXs that can be reached from the MixingTX 1 input contains CreateDenomTX A and CreateDenomTX B. Likewise, the set reachable from the MixingTX 2 input contains CreateDenomTX B and CreateDenomTX C. If we intersect the sets, CreateDenomTX B is the only CreateDenomTX remaining.

Modifications For actual transaction data, we do not know how many rounds users have been mixing for and whether the mixing originated from a single linkable address cluster AK18 GKRN18. Thus, we need to modify the attack. To compensate for not knowing how many rounds of mixing the inputs of a PrivateSendTX took, we consider a range of mixing rounds. To address the assumption that all inputs originated from a single cluster, we developed a two-fold approach.

First, we add a mechanism to reject a cluster if there is a subset of inputs that would result in another cluster. This cluster can be seen as an alternative explanation for the subset of inputs. The minimum size of the subset is adjustable via a parameter (alt). If, for example, ground-truth data indicated that clusters
containing $90 \%$ of the inputs are common, then an alt value of $80 \%$ could be suitable for blockchain analysts to safeguard the evidential value of their findings. In that case, the analysts would reject a cluster, if $80 \%$ or more of the inputs could be explained by another cluster.

Second, we add a mechanism to detect some obvious false positives that are based on the following observation. A cluster cannot have more Dash spent in its PrivateSendTXs than have been created in its CreateDenomTXs. This is why we compute a mix balance for each cluster as follows. Firstly, we sum the value of all outputs of CreateDenomTXs that are spent in MixingTXs. Then, we subtract the value of all inputs of a transaction that is not a MixingTX but is spending from MixingTXs. In simple terms, we determine the value that has been input into mixing and subtract the value that has been spent after mixing. Suppose the attack now suggests linking two clusters, such that the sum of their mix balances is negative. In that case, we know that we either encountered a false positive or that our clustering of pre-mix addresses was incomplete. In either case, we must reject the result because the two cases cannot be distinguished without ground-truth data.

This results in the Dash cluster-intersection attack (DC attack), which is a modified version of the algorithm proposed by Goldfeder et al. GKRN18. The algorithm is stated in Algorithm 1. The LEN method always returns the number of elements of the passed argument. The algorithm's input is a PrivateSendTX ptx and the parameter alt as described above. We set the starting value for the number of rounds $r$ to 2 , since 2 is the minimum number of mixing rounds required in Dash. The maximal possible number of rounds changed at the beginning of 2019 from 8 to 16 with protocol version 0.13.0.0 dasd]. Thus, to prevent the algorithm from detecting obvious false positives, we determine for every PrivateSendTX, the maxi-

```
Algorithm 1 DC attack
    procedure DC_ATTACK (ptx, alt)
        candidate \(=\) None
        \(r=2\)
        \(\max _{r}=\) DETERMINE_MAX_ROUNDS \((p t x)\)
        while \(r \leq \max _{r}\) do
            clusts \(=\emptyset\)
            for \(i n p \in p t x\).inps do
                clusts \([\) inp \(]=\) EXTRACT_CLUSTS \((r\), inp \()\)
            intersec \(=\bigcap_{\text {inp } \in \text { ptx. inps }}\) clusts \([\) inp \(]\)
            if \(\operatorname{LEN}(\) intersec \()==1\) then
                    if CORR_REJ(intersec, alt, clusts) then
                        candidate \(=\operatorname{GET}(\) intersec \()\)
                    break
        if BALANCE_CONF (candidate) then
            return candidate
        return None
```

            mal possible number of rounds \(\max _{r}\) in DETERMINE_MAX_ROUNDS as follows. We retrieve the block in which
            the transaction occurred. If the year extracted from the block's timestamp is greater than 2018 , we set \(\max _{r}\)
            to 16 and 8 otherwise. Next, the algorithm iterates over all rounds. In every round, for each input, all clusters
            that are attainable within \(r\) rounds of mixing are determined (EXTRACT_CLUSTS). Then, the intersection
            intersec of all found cluster sets is computed. If there is exactly one cluster in the intersection, we perform an
            additional check, CORR_REJ. This checks whether there is a subset of the inputs whose size is greater than or
            equal to alt of LEN(ptx.inps), which would lead to a different cluster than intersec. If this is not the case,
            candidate is set to the cluster in intersec (GET). The loop is left regardless of CORR_REJ. Finally, we check
            that candidate is not a false positive according to the mix balance, which is performed by BALANCE_CONF and works as explained above.
    The results of the DC attack are shown in Figure 6. Setting parameter alt to $100 \%$ corresponds to complete certainty in the assumption that all inputs resulted from one linkable cluster. In that case, no alternative explanation is taken into account and over $40 \%$ of the PrivateSendTXs are linkable. In the case of alt being equal to $0 \%$, all results would be rejected by definition.

$$
\begin{array}{r|rrrrrrrrrr}
\text { alt (\%) } & 10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 & 90 & 100 \\
\text { linkable (\%) } & 0.2 & 0.7 & 2.2 & 4.1 & 5.6 & 11.9 & 17.5 & 22.4 & 32.1 & 43.8
\end{array}
$$

Fig. 6: DC attack linkable PrivateSendTXs for parameter alt ranging from $10 \%$ to $100 \%$

Bitcoin Goldfeder et al. already demonstrated the applicability of the cluster-intersection attack in Bitcoin GKRN18. Thus, the crucial question in this work is whether there is on-chain information fueling the attack. The special vulnerability to cluster intersection in Dash results from the fact that users need to aggregate value in PrivateSendTXs. Thus, a PrivateSendTXs has several inputs from different MixingTXs in general that can be seen as such on-chain information.

To determine whether such on-chain information is also present in Bitcoin, we did the following. We checked for every PostCoinJoinTXs whether there are inputs from at least two different CoinJoinTXs. If this was the case, there was on-chain information as it is possible to intersect the anonymity sets of the different CoinJoinTXs. We found that out of the 7228843 PostCoinJoinTXs, 919532 (12.7\%) have inputs from at least two different CoinJoinTXs. This indicates that the coin aggregation problem is also present in Bitcoin.

## 5 Enhancing Privacy of Mixing

We show how to enhance the privacy of mixing and discuss direct countermeasures to mitigate the vulnerability to the Backlink attack. After discussing why fundamental changes to Dash seem unavoidable to prevent the DC attack, we propose a new mixing algorithm that removes the vulnerability to the cluster-intersection attack. This algorithm is of independent interest as it is not specific to Dash.

### 5.1 Preventing backlinks

The anonymity problems that come with backlinks are approachable within the design of Dash and Bitcoin. First, not all outputs of a CreateDenomTX must be input to MixingTXs. There may be change, such as discussed above in the example of Figure 4. Additionally, Dash allows for CoinControl, i.e. letting users in their wallet manually select inputs of a transaction dasf. While this is a useful feature, in the case of a user creating a BacklinkTX, we recommend explicitly warning them as backlinks remove the anonymity gained by mixing. A user's wallet should strictly separate any coins from CreateDenomTXs and those originating from a MixingTX. This idea is incorporated in the Bitcoin fungibility framework ZeroLink [zer] that distinguishes a pre-mix and a post-mix wallet. With version 0.16.0.1 dasc Dash improved its user interface following our recommendations after we disclosed our findings to them.

### 5.2 Cookie Monster Mixing

The vulnerability to the cluster-intersection attack results from the coin aggregation problem, that is the need to combine coins of different mixes. In Dash, this is a consequence of restricting the mixing to specific values. The logical solution would be to allow arbitrary values. However, arbitrary-value mixing suffers from privacy weaknesses caused by value analysis as discussed in Appendix B. Thus, we propose a new mixing algorithm, Cookie Monster Mixing. The basic principle behind Cookie Monster Mixing is to create a MixingTX where there are at least $k$ outputs with the same value and $k$ is the anonymity level the transaction should provide. This is related to the cookie monster problem BK15. Given a set of jars filled with various numbers of cookies, the cookie monster wants to eat all the cookies. However, the cookie monster has to proceed in rounds, select a subset of jars, and eat the same number of cookies from each jar in this subset. The goal is to eat the cookies in as few rounds as possible. In contrast, the objective in Cookie Monster Mixing is to maximize the number of cookies for a fixed number of rounds under constraints instead of minimizing rounds.

In Cookie Monster Mixing, a mixing service provider takes the role of the cookie monster, while the jars are inputs with a specific value of the cryptocurrency to be mixed. In Dash, the masternodes act as mixing service providers. Deviating from the cookie monster problem, let the number of rounds $r$ be fixed. In each round $j \in\{1, \ldots, r\}$, the mixing service provider may choose a target value $t_{j}$ and a subset of the input values from which $t_{j}$ is subtracted. The subtracted value is added to the output set, while the objective is to maximize the total output value. Intuitively that relates to maximizing the total value of anonymous coins. Additionally, there are two constraints. First, the size of the subset of inputs selected per round needs to be at least $k$. Second, each selected input needs to have a value at least as large as $t_{j}$. Together the constraints ensure that
the outputs determined via $t_{j}$ have at least an anonymity set size of $k$ as they have the same value and thus might have originated from any of the inputs selected in that round.

Integer Quadratic Problem The problem can be formulated as the following integer quadratic problem (IQP).

## Constants

$-v_{1}, \ldots, v_{n}$ : non-negative integers (values of $n$ inputs)
$-k$ : positive integer (minimum number of values to select per round)

## Variables

$-x_{1}, \ldots, x_{n}$ : 0-1 vectors of length $r$ (where $x_{i}$ denotes the rounds in which value $v_{i}$ has been selected)
$-t$ : non-negative integer vector of length $r$ (target values to be subtracted)

$$
\begin{align*}
& \text { maximise } \sum_{i=1}^{n}\left\langle x_{i}, t\right\rangle  \tag{1}\\
& \text { s.t. }\left\langle x_{i}, t\right\rangle \leq v_{i} \text {, for each } i \in\{1, \ldots, n\}  \tag{2}\\
& \qquad \sum_{i=1}^{n} x_{i, j} \geq k \text {, for each } j \in\{1, \ldots, r\} \tag{3}
\end{align*}
$$

The Objective (1) is to maximize the total anonymized value, while Constraints (2) and (3) ensure that it is actually possible to subtract $t_{j}$ from the selected inputs and that at least $k$ inputs are selected per round $j$ respectively.

Greedy Algorithm We propose a greedy algorithm that approximates the integer quadratic problem. It is stated in Algorithm 2. The input to the algorithm is a list of input values in_vals, as well as $k$ and $r$ as defined above. in_vals can be obtained from the multiset of inputs $I$ by replacing each input with its value. LEN returns the number of elements in a list and EXTRACT_K_LARGEST extracts the $k^{t h}$ largest element of a list. The algorithm returns target_vals, which is a list of values referring to the target values $t_{j}$ of the integer quadratic problem.

As long as the abort criterion (i.e., $\operatorname{LEN}($ in_vals $)<k \| r==0)$ is not fulfilled,

```
```

Algorithm 2 Greedy solver

```
```

Algorithm 2 Greedy solver
procedure SOLVER (in_vals, $k, r$ )
procedure SOLVER (in_vals, $k, r$ )
if $\operatorname{LEN}($ in_vals $)<k \| r==0$ then
if $\operatorname{LEN}($ in_vals $)<k \| r==0$ then
return $\emptyset$
return $\emptyset$
target_vals $=\emptyset$
target_vals $=\emptyset$
tmp_ $\bar{v}$ als $=\emptyset$
tmp_ $\bar{v}$ als $=\emptyset$
$r_{v a l}=$ EXTRACT_K_LARGEST $(k$, in_vals $)$
$r_{v a l}=$ EXTRACT_K_LARGEST $(k$, in_vals $)$
target_vals.add ( $r_{v a l}$ )
target_vals.add ( $r_{v a l}$ )
for val $\in$ in_vals do
for val $\in$ in_vals do
if $v a l>\bar{r}_{\_} v a l$ then
if $v a l>\bar{r}_{\_} v a l$ then
$t m p \_$vals.add $\left(v a l-r_{v a l}\right)$
$t m p \_$vals.add $\left(v a l-r_{v a l}\right)$
$t m p \_v a l s . a d d\left(v a l-r_{v}\right)$
else if $v a l<r_{\text {_ }}$ val then
$t m p \_v a l s . a d d\left(v a l-r_{v}\right)$
else if $v a l<r_{\text {_ }}$ val then
tmp_vals.add(val)
tmp_vals.add(val)
return target_vals.concat(S O LVER(
return target_vals.concat(S O LVER(
$\left.t m p \_v a l s, k, r-1\right)$ )

```
```

                        \(\left.t m p \_v a l s, k, r-1\right)\) )
    ```
```

of in_vals, which is assigned to $r_{v a l}$. Since this element can be seen as the target value of the greedy algorithm in that round, it is added to target_vals. Then, the input values are updated as follows. In case a value is greater than $r_{v a l}$, their difference is added to tmp_vals. Otherwise, if the value is smaller than $r_{v a l}$, the value itself is added to $t m p_{\_} v a l$. If they are equal, the value is omitted. This behavior corresponds to inherently selecting all possible inputs per round in terms of the integer quadratic problem. The algorithm runs in polynomial time. There are at most $r$ recursive calls and the runtime of each call is mainly determined by the time it takes to extract the $k^{t h}$ largest element of a list. If this is implemented by sorting, the algorithm runs in $\mathcal{O}(r \cdot n \log n)$.

Our greedy algorithm is not optimal, which can be seen by the following example. Let in_vals = $[2,2,1], k=2$ and $r=2$. In the first round, the algorithm extracts 2 as the second largest element and adds
it to target_vals, while 1 is added to tmp_vals. In the recursive call, $\emptyset$ is returned, as the length $i n_{\sim}$ _vals is 1 and thus smaller than $k$ which satisfies the abort criteria. Consequently, the output value achieved in terms of Objective (1) is only 4 , as target_vals $=[2]$ and the greedy algorithm inherently selects all possible inputs per round, which are the first two of in_vals in the first round. The optimum 5 , however, is achieved by setting $a_{1}=a_{2}=x_{1,1}=x_{2,1}=x_{3,1}=x_{1,2}=x_{2,2}$ to 1 and $x_{3,2}$ to 0 .

Evaluation of Algorithm 2 To measure the quality of our greedy algorithm in terms of maximizing Objective (1), we evaluated it against the optimal solution. Therefore we modelled the integer quadratic problem as given by Objective (1) and Constraints (2) and (3) in IBM's Optimization Programming Language (OPL) and used the mixed integer optimizer from IBM LOG CPLEX Optimization Studio V12.10.0 ibm.

For the values of the inputs (in_vals), we first considered all clusters of CreateDenomTXs retrieved by applying the multi-input heuristic and Heuristic 41 as discussed in Section 4.2. For each cluster, we summed up the values of all outputs that were referenced by MixingTXs. This results in a distribution of Dash that users were mixing in the past. It is therefore better suited for evaluation than purely random values. We varied both, $r$ and the number of inputs that we chose randomly from the distribution. We set $k$ to 3 following the Dash whitepaper, which suggests at least three participants per mixing round [DD. The results are shown in Figure 7 averaged over 100 runs indicating that Algorithm 2 is nearly as good as the optimal solution. The average wall-clock time of the solver is reported in Figure 7b. In contrast, Algorithm 2 took less than a second for each choice of parameters. Therefore, particularly for multiple inputs and rounds, using a solver is infeasible, which is why Algorithm 2 should be used instead.

|  | $r$ |  |  |
| :---: | :---: | :---: | ---: |
| $\mid$ in_vals | 3 | 4 | 5 |
| 5 | $98.8 \%$ | $98.5 \%$ | $98.1 \%$ |
| 10 | $98.2 \%$ | $98.4 \%$ | $98.3 \%$ |
| 15 | $97.0 \%$ | $97.2 \%$ | $98.3 \%$ |
| 20 | $96.1 \%$ | $95.8 \%$ | $97.7 \%$ |

(a) Avg. performance of Algorithm 2 w.r.t Objective (1)

|  | $r$ |  |  |
| :---: | ---: | ---: | ---: |
| $\mid$ in_vals $\mid$ | 3 | 4 | 5 |
| 5 | 0.02 | 0.02 | 0.28 |
| 10 | 0.03 | 0.27 | 7.80 |
| 15 | 0.07 | 7.51 | 68.71 |
| 20 | 0.15 | 5.98 | 180.14 |

(b) Avg. wall-clock time of optimizer in minutes

Fig. 7: Evaluation of Algorithm 2 against optimizer

Using Cookie Monster Mixing removes the need for splitting and aggregating coins before and after mixing. Thus, the on-chain information that multiple coins of different mixes belong together is no longer present, preventing the DC attack. To prevent cluster-intersection attacks from off-chain information as well, Cookie Monster Mixing needs to be combined with privacy-aware wallets and browsers.

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## References

AK18. Antti-Kaikkonen. Evaluating the privacy of privatesend, 2018.
$\mathrm{AKR}^{+}$13. Elli Androulaki, Ghassan Karame, Marc Roeschlin, Tobias Scherer, and Srdjan Capkun. Evaluating user privacy in Bitcoin. In Sadeghi Sad13, pages 34-51.
$\mathrm{BCG}^{+}$14. Eli Ben-Sasson, Alessandro Chiesa, Christina Garman, Matthew Green, Ian Miers, Eran Tromer, and Madars Virza. Zerocash: Decentralized anonymous payments from bitcoin. In 2014 IEEE Symposium on Security and Privacy, pages 459-474. IEEE Computer Society Press, May 2014.
bit. Bitcoin.
BK15. Leigh Marie Braswell and Tanya Khovanova. The cookie monster problem. The Mathematics of Various Entertaining Subjects: Research in Recreational Math, page 231, 2015.
bloa. Blocksci source code.
Blob. Blockstream. Bitcoin explorer source code.
btc. Bitcoin release v0.20.0.
dasa. Dash.
dasb. Dash core repository.
dasc. Dash core repository release history.
dasd. Dash core version 0.13.0.0.
dase. Dash developer documentation.
dasf. Dash wallet documentation - coin control.
dasg. Dash wallet documentation - privatesend and instantsend.
DD. Evan Duffield and Daniel Diaz. Dash: A payments-focused cryptocurrency.
Eur. European Cybercrime Center (EC3). Internet organised crime threat assessment 2020.
GKRN18. Steven Goldfeder, Harry Kalodner, Dillon Reisman, and Arvind Narayanan. When the cookie meets the blockchain: Privacy risks of web payments via cryptocurrencies. Proceedings on Privacy Enhancing Technologies, 2018(4):179-199, 2018.
ibm. Ibm log cplex optimization studio v.12.10.0.
Ike. Scott Ikeda. South korea's new crypto aml law bans trading of "privacy coins" (monero, zcash).
joi. Joinmarket.
KFTS17. Amrit Kumar, Clément Fischer, Shruti Tople, and Prateek Saxena. A traceability analysis of monero's blockchain. In Simon N. Foley, Dieter Gollmann, and Einar Snekkenes, editors, ESORICS 2017, Part II, volume 10493 of LNCS, pages 153-173. Springer, Heidelberg, September 2017.
$K_{G C}{ }^{+}$17. Harry Kalodner, Steven Goldfeder, Alishah Chator, Malte Möser, and Arvind Narayanan. Blocksci: Design and applications of a blockchain analysis platform. arXiv preprint arXiv:1709.02489, 2017.
$K_{M L}{ }^{+}$20. Harry A. Kalodner, Malte Möser, Kevin Lee, Steven Goldfeder, Martin Plattner, Alishah Chator, and Arvind Narayanan. BlockSci: Design and applications of a blockchain analysis platform. In Srdjan Capkun and Franziska Roesner, editors, USENIX Security 2020, pages 2721-2738. USENIX Association, August 2020.

KYMM18. George Kappos, Haaroon Yousaf, Mary Maller, and Sarah Meiklejohn. An empirical analysis of anonymity in zcash. In William Enck and Adrienne Porter Felt, editors, USENIX Security 2018, pages 463-477. USENIX Association, August 2018.
Max13. Gregory Maxwell. Coinjoin: Bitcoin privacy for the real world, 2013.
MM18. Sarah Meiklejohn and Rebekah Mercer. Möbius: Trustless tumbling for transaction privacy. PoPETs, 2018(2):105-121, April 2018.
MNF17. F. K. Maurer, T. Neudecker, and M. Florian. Anonymous coinjoin transactions with arbitrary values. In 2017 IEEE Trustcom/BigDataSE/ICESS, pages 522-529, Aug 2017.
mon. Monero.
$\mathrm{MPJ}^{+}$13. Sarah Meiklejohn, Marjori Pomarole, Grant Jordan, Kirill Levchenko, Damon McCoy, Geoffrey M Voelker, and Stefan Savage. A fistful of bitcoins: characterizing payments among men with no names. In Proceedings of the 2013 conference on Internet measurement conference, pages 127-140. ACM, 2013.
$\mathrm{MSH}^{+}$18. Malte Möser, Kyle Soska, Ethan Heilman, Kevin Lee, Henry Heffan, Shashvat Srivastava, Kyle Hogan, Jason Hennessey, Andrew Miller, Arvind Narayanan, and Nicolas Christin. An empirical analysis of traceability in the monero blockchain. PoPETs, 2018(3):143-163, July 2018.
NML16. Shen Noether, Adam Mackenzie, and The Lab. Ring confidential transactions. Ledger, 1:1-18, 122016.
Que17. Jeffrey Quesnelle. On the linkability of Zcash transactions. arXiv e-prints, page arXiv:1712.01210, Dec 2017.

RMK14. Tim Ruffing, Pedro Moreno-Sanchez, and Aniket Kate. CoinShuffle: Practical decentralized coin mixing for bitcoin. In Miroslaw Kutylowski and Jaideep Vaidya, editors, ESORICS 2014, Part II, volume 8713 of $L N C S$, pages 345-364. Springer, Heidelberg, September 2014.
RMK17. Tim Ruffing, Pedro Moreno-Sanchez, and Aniket Kate. P2P mixing and unlinkable bitcoin transactions. In NDSS 2017. The Internet Society, February / March 2017.
RS13. Dorit Ron and Adi Shamir. Quantitative analysis of the full Bitcoin transaction graph. In Sadeghi Sad13, pages 6-24.
Sad13. Ahmad-Reza Sadeghi, editor. FC 2013, volume 7859 of LNCS. Springer, Heidelberg, April 2013.
sam. Samourai wallet.
VS13. Nicolas Van Saberhagen. Cryptonote v 2.0, 2013.
was. Wasabi wallet.
zca. Zcash.
zer. Zerolink - the bitcoin fungibility framework.

## A Differences in the Analysis in Bitcoin

This section highlights the general differences in the analysis of Bitcoin and Dash. For Bitcoin, we ran a full node, version 0.20.0 [btc and used BlockSci $\left[\mathrm{KML}^{+} 20\right]$ with version 0.7.0. We retrieved the raw blockchain data corresponding to a chain of 612793 blocks with 493118000 transactions.

The main difference occurs in the transaction type detection as Bitcoin neither knows CreateDenomTXs nor PrivateSendTXs. Thus, besides the CoinJoinTXs, we also consider PreCoinJoinTXs and PostCoinJoinTXs. A PreCoinJoinTX is any transaction that has at least one output being referenced by a CoinJoinTX. Likewise, a PostCoinJoinTX is any transaction referencing at least one output of a CoinJoinTX. We further exclude all CoinJoinTXs from PreCoinJoinTXs and PostCoinJoinTXs. In comparison with Dash, the CoinJoinTXs would correspond to MixingTXs, the PreCoinJoinTXs to the CreateDenomTXs, and the PostCoinJoinTXs to the PrivateSendTXs. Detecting PostCoinJoinTXs and PreCoinJoinTXs is straightforward once CoinJoinTXs are detected. However, the detection of CoinJoinTXs is difficult as there are multiple different CoinJoin services in Bitcoin (e.g. [joi]was/sam]) that neither require the number of inputs and outputs to be the same nor restrict their inputs to specific denominations as is the case in Dash. BlockSci already implements a CoinJoin detection mechanism which, however, is tailored to JoinMarket [joi] transactions and therefore does not recognize the transactions of other CoinJoin services such as Wasabi Wallet [was] or Samurai Wallet [sam]. For this reason, we adapted the CoinJoin detection mechanism of the opensource Blockstream Bitcoin explorer [Blob] as it is capable of detecting CoinJoinTXs of several services. However, we also adopted the elements of BlockSci's algorithm bloa that were not specific to JoinMarket. Our algorithm is stated in Algorithm 3. The algorithm returns True if the provided transaction $t x$ is (most likely) a CoinJoin transaction. The LEN method returns the number of elements of the passed argument, MIN and MAX work as expected. OCC computes the number of occurrences of the value val in the provided outputs. OCC_MOST returns the number of occurrences of the value that occurs the most while OCC_UNIQ returns the number of unique output values. The first thing the algorithm does is check

```
Algorithm 3 CoinJoin detection
```

Algorithm 3 CoinJoin detection
procedure COJO_DETECTION $(t x)$
procedure COJO_DETECTION $(t x)$
if $\operatorname{LEN}($ tx.inps $)<\overline{2} \| \operatorname{LEN}(t x$.outs $)<3$ then
if $\operatorname{LEN}($ tx.inps $)<\overline{2} \| \operatorname{LEN}(t x$.outs $)<3$ then
return False
return False
target $=\operatorname{MIN}(\operatorname{MAX}(\operatorname{LEN}($ tx.outs $) / 2,2), 5)$
target $=\operatorname{MIN}(\operatorname{MAX}(\operatorname{LEN}($ tx.outs $) / 2,2), 5)$
found $=$ False
found $=$ False
for out $\in$ tx.outs do
for out $\in$ tx.outs do
if out.val $==546 \|$ out.val $==2730$ then
if out.val $==546 \|$ out.val $==2730$ then
return False
return False
if $\operatorname{OCC}($ val, tx.outs $)>=$ target $)$ then
if $\operatorname{OCC}($ val, tx.outs $)>=$ target $)$ then
found $=$ True
found $=$ True
if OCC_MOST $($ tr.outs $)<$ OCC_UNIQ(tx.outs) then
if OCC_MOST $($ tr.outs $)<$ OCC_UNIQ(tx.outs) then
return False
return False
return found

```
        return found
```

two inputs and three outputs. The reason for this is that mixing requires at least two participants. At least one participant generally receives some change, which is why there is always at least one additional output aside from the two mixed ones. Next, a target between two and five is computed. This is used to check whether
there are at least target many outputs with the same value, where target corresponds to half the number of outputs but is kept between two and five as done by Blockstream Blob. As suggested by BlockSci, a transaction with so-called dust outputs is unlikely a CoinJoin transaction, which is why output values should not be equal to 546 or 2730 bloa. These values are the smallest possible output values allowed by Bitcoin Core depending on the version. The last check is to prevent false positives as there needs to be at least as many equal-valued outputs as there are unique ones. The reason is that in a CoinJoinTX unique outputs should only be change outputs.

## B Limitations to Arbitrary-Value Mixing



Fig. 8: CoinJoin transaction
We proposed Cookie Monster Mixing (see Section 5.2) as arbitrary-value mixing is not a suitable solution for the coin aggregation problem due to privacy weaknesses based on value analysis. When mixing coins with an arbitrary value, outputs can usually be linked to the corresponding inputs by inspecting the values, as discussed by Maurer et al. MNF17. Considering the CoinJoin transaction in Figure 8a taken from Maurer et al. MNF17, the transaction can only consist of the two sub-transactions (dotted line and dashed line), such that it is possible to link inputs $i_{1}$ and $i_{2}$ to outputs $o_{1}$ and $o_{2}$, as well as $i_{3}$ and $i_{4}$ to $o_{3}$ and $o_{4}$, respectively. To prevent this linkage, Maurer et al. [MNF17] propose output-splitting algorithms. Given two transactions, their basic splitting algorithm works by calculating the difference between the sums of the corresponding output lists. Next, one of the outputs of the list with the larger sum is split to produce this difference [MNF17. Thus, multiple input-output relations are possible. Applied to the two sub-transactions in Figure 8a, the algorithm results in the transaction depicted in Figure 8b. Output $o_{3}$ in Figure 8a has been split in $o_{3.1}$ and $o_{3.2}$ such that $i_{1}$ and $i_{2}$ belong to either $o_{1}$ and $o_{2}$ or $o_{3.2}$ and $o_{4}$. The reason is that the sum of the values of $i_{1}$ and $i_{2}$ equals 33 , as do the sums of $o_{1}$ and $o_{2}$ as well as $o_{3.2}$ and $o_{4}$.

However, even if output-splitting results in multiple potential input-output relations, it is still possible to determine the actual input-output relation by inspecting the values. In Figure 8b, $i_{1}$ and $i_{2}$ are far more likely to result in $o_{1}$ and $o_{2}$ than in $o_{3.1}$ and $o_{4}$. The reason is that under the assumption that one of the outputs is change, input $i_{2}$ would not have been required as $i_{1}$ is larger than $19\left(o_{3.1}\right)$ and $14\left(o_{4}\right)$.

As their basic output-splitting algorithm does not affect the input linkability, that is $i_{1}$ and $i_{2}$ as well as $i_{3}$ and $i_{4}$ are linkable, Maurer et al. MNF17] propose a version of the algorithm that implements input shuffling. Instead of using the difference between the sums of the corresponding output lists, the sum of a random subset of inputs is used to split the outputs. Thereby, the number of inputs is a parameter of the algorithm. In terms of Figure 8a, the input shuffling algorithm might employ the sum of $i_{1}, i_{2}$ and $i_{4}$. While this seems to be an improvement over the basic algorithm, it is especially dangerous if the inputs are linkable by heuristics. Intuitively, the gained privacy relies on an ambiguity on the input side, which is introduced by using the sum of a random subset in output splitting. However, if it is known which inputs belong together, there is no gain in privacy at all.


[^0]:    ${ }^{1}$ Note that the reference says $\left[\right.$ denom $\left._{\text {hash }}, 3\right]$ to refer to the fourth output as indexing starts with 0 .

